

The Direction of Gravity

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Abstract. Gravity directs the paths of light rays and the growth of structure. Moreover, gravity on cosmological scales does not simply point down: it accelerates the universal expansion by pulling outward, either due to a highly negative pressure dark energy or an extension of general relativity. We examine methods to test the properties of gravity through cosmological measurements. We then consider specific possibilities for a sound gravitational theory based on the Galileon shift symmetry. The evolution of the laws of gravity from the early universe to the present acceleration to the future fate – the paths of gravity – carries rich information on this fundamental force of physics and on the mystery of dark energy.

1. Introduction

The direction of gravity, in both senses of the word, is a key characteristic of the universe. The revolution of the late 1990s was the discovery that gravity did not exclusively pull down in the conventional, attractive way but that cosmic expansion showed an acceleration equivalent to gravity pulling outward. Secondly, gravitation plays the major role in directing the contents of the universe, bending the paths of light and governing the growth of large scale structure.

A new frontier has opened in exploring the force of gravity, understanding not only its spatial direction but its direction of energy and mass in the sense of a conductor directing an orchestra. Some outstanding questions now being addressed include: Why is gravity pulling outward – due to an unknown dark energy component or a new law? Is gravity (e.g. Newton’s constant) constant, or strengthening or weakening with time? Does gravity govern the growth of cosmic structure in exactly the manner it governs cosmic expansion, as in the laws of general relativity, or are there further degrees of freedom? Does gravity behave the same on all scales?

At its most basic, we should test our understanding of cosmic gravity because we can, as a reality check. Observations of the deflection of light by gravity (lensing) and the growth of matter clustering by gravity are reaching the stage where they can provide incisive information. Moreover, we should justify the long extrapolation of general relativity from the small scales where it is tested to the cosmic scales where it is applied (more than a factor of billion in length), and from high curvature to low curvature regions. Tests of cosmic gravity are further motivated by the fact that the first two precision observational tests have basically shown gaping holes in our understanding: general relativity plus attractive matter fails to predict acceleration in the cosmic expansion, and general relativity plus attractive matter fails to agree with the growth and clustering of large scale structure.

In Sec. 2 we explore methods for testing gravity in phenomenological and model independent approaches. Adopting a specific model showing desirable characteristics, Galileon gravity, in

Sec. 3 we illustrate the theoretical constraints an alternate theory of gravity must satisfy and the handles by which future data will be able to test stringently the nature of gravity.

2. The Differences of Gravity

To test general relativity one can either simply look for consistency with observations or one can measure parameters or quantities that would differ from the general relativity prediction if deviations exist. For an early work addressing some implications of deviations, e.g. from the inverse square law of attraction, see [1]. We will concentrate here on cosmic scales, those above galaxy cluster lengths but well within the horizon, 5–500 Mpc.

General relativity says that effects on the cosmic expansion are mirrored exactly in the growth of structure, with no extra gravitational degrees of freedom to cause an offset. Therefore comparing the expansion history to the growth history is one of the major tests of the physics. This implies that it is crucial to fit for the expansion and growth simultaneously, to be alerted to any deviations. Should the gravity theory be assumed (e.g. as general relativity) incorrectly, this will bias the fits of both the expansion and cosmological model. The converse is true as well (e.g. assuming Λ CDM). Thus, even if one is not interested in gravity one still must fit for it or the quantities one is interested in will be biased.

To separate cleanly the gravity effects on growth from the expansion effects on growth, the gravitational growth index parameter γ was developed by [2]. Recall that linear growth of a fractional density perturbation $\delta\rho/\rho$ as a function of expansion factor a can be well approximated by

$$\frac{\delta\rho}{\rho}(a) = \frac{\delta\rho}{\rho}(a_i) \times e^{\int_{a_i}^a (da'/a') \Omega_m^\gamma(a')} \quad (1)$$

$$\Omega_m(a) = \frac{\Omega_m a^{-3}}{\Omega_m a^{-3} + (1 - \Omega_m) e^{3 \int_a^1 (da'/a') [1+w(a')]}}, \quad (2)$$

where Ω_m is the present matter density in units of the critical density and the dark energy equation of state $w(a) = w_0 + w_a(1 - a)$ to an excellent approximation. This accurately deconvolves the two influences on the growth, the expansion history in the form of $\Omega_m(a)$ from the gravity law in the form of γ , providing a fit to better than 0.1% in many of the standard cases. Suppose one considered growth measurements arising from weak gravitational lensing surveys. The weak lensing signal involves a convolution of the gravitational growth of massive structures at various redshifts along the line of sight and the geometric distance factors serving as a focal length. If one assumes general relativity (GR), omitting to fit for gravity, then the dark energy parameters one derives from the (GR) growth and distances could be strongly biased. The fit for the time variation of the dark energy equation of state w_a will be incorrect by $\Delta w_a \approx 8\Delta\gamma$, where $\Delta\gamma$ is the amount by which the true gravity deviates from GR. That is, failing to fit for gravity also induces failure to fit expansion accurately. The converse is true as well. Fitting for gravity is therefore necessary.

If we want to go beyond the coarse grained approach of γ , how should we do so in a practical manner? Adding parameters without careful thought will merely blow up the uncertainties in both gravity and expansion. As with the gravitational growth index γ , one must carefully study how to separate cleanly the different physical influences. One could work within a specific model that determines the gravity and expansion behaviors but such one by one comparisons are time consuming and often give little general insight. Instead we consider various approaches to a more model independent analysis.

General relativity, as summarized by John Wheeler, is that “matter tells spacetime how to curve, and spacetime tells matter how to move”. These statements can be thought of mathematically as Poisson’s equation $\nabla^2\phi = 4\pi G_N a^2 \delta\rho$ and Newton’s first law $-\vec{\nabla}\psi = \ddot{x}$.

We have suggestively written the spacetime potentials with two different symbols: ϕ and ψ . In GR they are one and the same, but in other theories of gravity they can differ. To tie them closely to the observations we consider two modified Poisson equations,

$$\nabla^2(\phi + \psi) = 8\pi G_N a^2 \delta\rho \times G_{\text{light}} \quad (3)$$

$$\nabla^2\psi = 4\pi G_N a^2 \delta\rho \times G_{\text{matter}} . \quad (4)$$

The function G_{light} tests how light responds to gravity, and is central to gravitational lensing and integrated Sachs-Wolfe measurements. The function G_{matter} tests how matter responds to gravity, and is central to growth of massive structures and peculiar velocities; the growth index γ is closely related. Also see [3] for a more detailed discussion of these modified Poisson equations.

In general G_{light} and G_{matter} will be functions of time and space, e.g. redshift $z = a^{-1} - 1$ and length scale r or wavemode k . One approach to parametrizing them with a practical number of degrees of freedom is to be guided by classes of gravity theories. Looking at both higher dimension gravity and scalar-tensor theories one sees a deviation starting from early universe GR behavior and involving either a scale independent or characteristic k^2 dependence (arising from the Laplacian in the equations of motion). Thus, a useful form is [4]

$$G_{\text{matter}} = 1 + \frac{ca^s(k/H_0)^n}{1 + 3|c|a^s(k/H_0)^n} , \quad (5)$$

where $n = 0, 2$ respectively and a similar form could hold for G_{light} . Note that leaving out the denominator, so G simply deviates from the GR value of 1 as a^s is dangerous as it unfairly weights high vs low redshift observations and can easily lead to bias. The Padé form of Eq. (5) is accurate to $\sim 1\%$ for DGP and $f(R)$ gravity.

For current data, involving weak gravitational lensing, galaxy peculiar velocities, CMB power spectra, supernova distances, and baryon acoustic oscillations, Ref. [4] simultaneously fits for gravity (the amplitude c and time dependence s) and expansion (matter density Ω_m and dark energy equation of state w_0, w_a). They find the gravity constraints are nearly unaffected by the expansion fit, and the expansion constraints are nearly unaffected by the gravity fit. To a significant extent, however, this reflects the status of current data: almost no constraint exists on s . As data improves and becomes more incisive in testing gravity we do expect some degradation in expansion constraints when also fitting, vs fixing, gravity – perhaps a factor of ~ 2 in confidence contour area.

Both the above theories of gravity mentioned are simple models with restricted parameters. DGP [5] has one gravity parameter, the 5D crossover scale that determines both the amplitude and time dependence of the deviations from GR. The $f(R)$ family of scalar-tensor models has more freedom but can successfully be treated in terms of the scalaron mass [6], and this is well fit by an amplitude (of order $\sim 100H_0$) and a power law time dependence (i.e. s) – basically two parameters.

Indeed, if one considers the phase space, or “paths of gravity”, of the deviations from GR, DGP shows a definite, near parabolic track in the $G'_{\text{matter}}-G_{\text{matter}}$ plane and the family of $f(R)$ tracks can be calibrated through a rescaling of s into a single, also near parabolic track [7]. See Figure 3 of [7] for an illustration and further details. These two classes diverge from each other, and GR, having today $\Delta G_{\text{matter}} \approx \pm 0.3$. This suggests, in these cases at least, that we should strive for observations capable of testing this beyond Einstein parameter at the 10% level or better.

Not every viable theory of gravity may be so simple, however. It is useful to have another, more model independent approach in our toolkit. A reasonable choice is to test for the values of the G functions in bins of redshift and wavenumber, i.e. asking whether gravity behaves the same (and like GR) at high and low redshift, and for high and low wavenumbers (small and

large scales). While basis functions or principal components are other possible choices, even next generation data will not have the leverage to fit more than 2 modes each with reasonable signal to noise (not just low noise). Recall that N modes in each of redshift and wavenumber gives rise to $2N^2$ parameters (for G_{matter} and G_{light}), and hence $N^2(2N^2 + 1) \sim 2N^4$ correlation functions, so attempting to fit more than 2 modes is impractical. We thus consider “ $2 \times 2 \times 2$ ” gravity: high/low redshift, large/small scales, G_{matter} and G_{light} [8].

Such an approach is fully model independent and we can investigate the leverage of current and future observations to test cosmic gravity. In Figure 1 we show the results of Markov Chain Monte Carlo fits to data and simulations in the two bins each of wavenumber and redshift. Galaxy redshift surveys such as BigBOSS [9] will be powerful “gravity machines”, capable of mapping the density and velocity fields over a wide range of redshifts. This is particularly useful at constraining G_{matter} , the function sensitive to growth. Gravitational lensing information and the integrated Sachs-Wolfe effect of the cosmic microwave background act to constrain G_{light} .

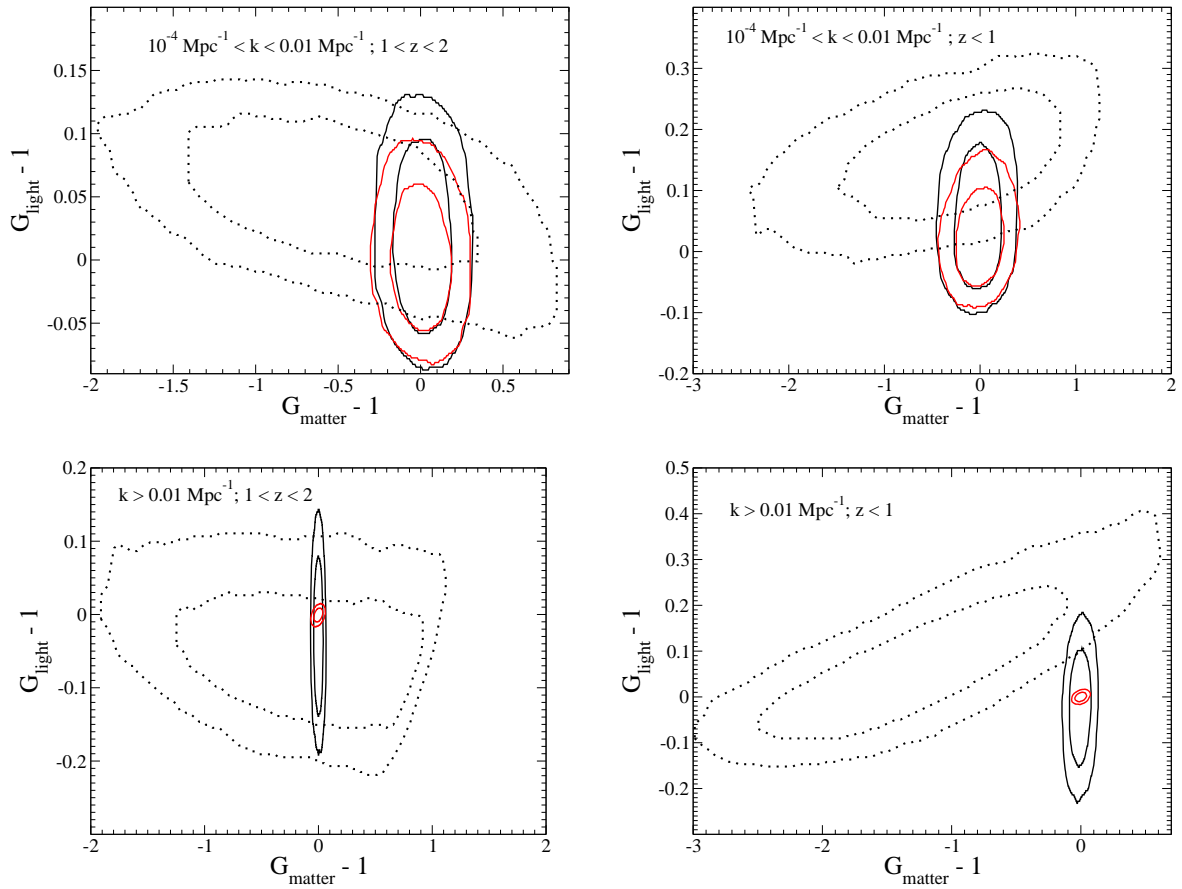


Figure 1. Constraints are shown on the 8 parameters of post-GR model independent gravity, from current and simulated future data. Dotted contours show the 68% and 95% confidence regions from current WMAP CMB [10], CFHTLS lensing [11], and Union2.1 supernova distance data [12], while solid black contours show constraints from simulated data of the next generation BigBOSS galaxy redshift survey (marginalized over galaxy bias) plus Stage III experiments. Small red contours include as well a highly optimistic galaxy weak lensing survey covering 4000 deg^2 with a galaxy number density of 55 arcmin^{-2} .

We see that current data are weak in the ability to test G_{matter} , with uncertainties of order one, while G_{light} can be determined to roughly 10% from the Canada-France-Hawaii Telescope Legacy Survey lensing data [11] and the WMAP CMB temperature power spectrum [10]. However, note that this precision does not indicate accuracy; the deviation from GR seen in the bottom right panel, for example, arises from acknowledged observational systematics [8]. Galaxy redshift surveys such as BigBOSS will bring G_{matter} also to the 10% precision level. Recall that this is just what the “paths of gravity” phase space approach above argued was necessary to test GR, so we anticipate that next generation observations will reach interesting leverage in understanding cosmic gravity.

For the galaxy surveys one must marginalize over astrophysical effects such as galaxy bias. Here we have taken care of that by allowing the bias to float freely in each 0.1 bin of redshift. Due to the complementarity with the other techniques included in Stage III experiments, this marginalization causes little degradation ($\lesssim 10\%$) in the final constraints. To reduce the uncertainties on G_{matter} further, one would need to increase the survey volume or use other growth probes such as CMB lensing. To show the effect of a next generation galaxy weak lensing survey on G_{light} , we include a rather optimistic survey in the inner red contours, with the result that G_{light} could be determined in some bins at the 1% level. In any case, the future of testing gravity looks extremely promising.

3. The Paths of Gravity

Exploring beyond the simplest models of gravity is useful to understand what evolutionary signatures the gravitational modifications might exhibit. This allows us to investigate how robust are the well defined phase space tracks predicted by those particular models, and to look for characteristics that might be smoothed over by the model independent binned approach of the previous section.

Moreover, $f(R)$ models are arbitrary and unnatural in the same sense that a quintessence potential $V(\phi)$ is: we have no clear physics guidance as to the appropriate functional form and high energy physics corrections will in any case alter it. Two ways to protect against these problems are to avoid completely any potential, using only kinetic terms, and to look for a theory where the degrees of freedom arise geometrically, e.g. from higher dimensions, protecting against corrections. Galileon gravity incorporates both these solutions.

Galileon gravity is based on a shift symmetry in the effective field π and involves nonlinear kinetic terms that are allowed (and only these are allowed) in order to guarantee second order field equations [13, 14]. These also provide a Vainshtein screening that restores the theory to general relativity on small scales. Thus, this is a well defined, robust theory that we can explore as a possible extension to GR.

We can write the action for the Galileon theory as

$$\begin{aligned}
S = \int d^4x \sqrt{-g} & \left[\left(1 - 2c_0 \frac{\pi}{M_{\text{pl}}} \right) \frac{M_{\text{pl}}^2 R}{2} - \frac{c_2}{2} (\partial\pi)^2 - \frac{c_3}{M^3} (\partial\pi)^2 \square\pi - \frac{c_4 \mathcal{L}_4}{2} - \frac{c_5 \mathcal{L}_5}{2} \right. \\
& \left. - \frac{M_{\text{pl}}}{M^3} c_G G^{\mu\nu} \partial_\mu \pi \partial_\nu \pi - \mathcal{L}_m \right]. \quad (6)
\end{aligned}$$

General relativity arises simply from the “1” inside the square brackets. The terms involving c_2 through c_5 comprise the standard Galileon extension [13, 14], and one can consider additional couplings to matter given by the c_0 term for a linear coupling and c_G for a derivative coupling. Again, these terms guarantee second order field equations.

From the action one can derive the background equations of motion and find under what circumstances one gets late time acceleration. Note that acceleration occurs despite the absence of any cosmological constant or indeed any scalar field potential. Galileon cosmology has

attractor solutions in the radiation and matter eras, avoiding fine tuned sensitivities to initial conditions. The effective equation of state w at early times can be close to that of matter, allowing for a nonnegligible contribution of early dark energy density. The field generically becomes phantom, $w < -1$, at some epoch near today, and has a future attractor to a de Sitter state. These are all interesting properties for a theory to possess, and are discussed in some detail in [15].

Perturbing the equations of motion leads to the modified Poisson equations for G_{matter} and G_{light} , and the growth of structure. We can then study the paths of gravity, the evolution of these $G(a)$ functions. In the early universe Galileon gravity acts as a thawing field, locked to general relativity at high redshift and then gradually deviating. The deviation increases linearly as the effective dark energy density does: $G \sim 1 + b\Omega_\pi$. However, unlike the DGP and $f(R)$ cases, the evolution is not a simple, quasi-parabolic trajectory from GR to a frozen new value of the gravitational strength. The interaction between the multiple Galileon terms leads to nonmonotonic behavior, strengthening gravity at redshift $z \sim 10$ then restoring to GR before deviating again toward a de Sitter attractor. Interestingly, in this asymptotic state the gravitational slip, the difference between the metric potentials ϕ and ψ with which we started our exploration of extensions to GR, vanishes (and hence $G_{\text{matter}} = G_{\text{light}}$). But although $\phi = \psi$ this is not general relativity, i.e. the strength of gravity differs from Newton's constant, $G \neq 1$.

Galileon cosmology thus shows some fascinating properties and signatures by which we can hope to distinguish it observationally from GR. Figure 2 shows the paths of gravity, the evolution of $G_{\text{matter}}(a)$ and $G_{\text{light}}(a)$ for the uncoupled Galileon for different values of the initial dark energy density. The larger the initial density, the stronger the deviation from GR at $z \sim 10$ and the closer to the present the bump occurs; a low initial density is still capable of giving the same late time behavior but with an earlier and milder first deviation from GR. One might conjecture whether the early enhancement in the gravitational strength could play a role in the development of high redshift structure and early massive galaxy clusters.

Detailed comparison of the predictions of Galileon cosmology for the expansion and growth history with observational data is in progress [16]. However, physical soundness of the theory in terms of lacking pathologies or instabilities already puts important constraints on the Galileon parameters [15]. The most important conditions are the no-ghost condition, preventing the energy from being unbounded from below, and stability of perturbations, which can be written in terms of the sound speed as $c_s^2 \geq 0$. These requirements not only restrict the viable parameter space but in the linear and derivative coupling cases force those contributions to be subdominant to the standard Galileon terms at high redshift.

Galileon cosmology has a much richer phenomenology than the simple extensions to GR previously considered and we are still exploring its implications. It has a sounder theoretical foundation than many other theories (including most quintessence models) and the combination of theoretical and observational constraints may soon give rise to definite predictions for how to find deviations from general relativity.

4. Conclusions

Gravitation is a ubiquitous and fundamental force but one that has not yet been rigorously tested, especially over the immense extrapolation to cosmic scales. The “direction of gravity” is an open, active area of research and may hold key insights into cosmic acceleration and the growth of large scale structure.

Measuring the expansion history alone, i.e. the dark energy equation of state $w(a)$, is not sufficient to reveal the physics; rather, the combination of the expansion history and growth history, or testing gravity as well as the equation of state, is essential. This introduces the functions G_{matter} and G_{light} from the modified Poisson equations, detailing gravity's direction of matter growth and of light deflection. Failure to fit simultaneously for expansion and gravity

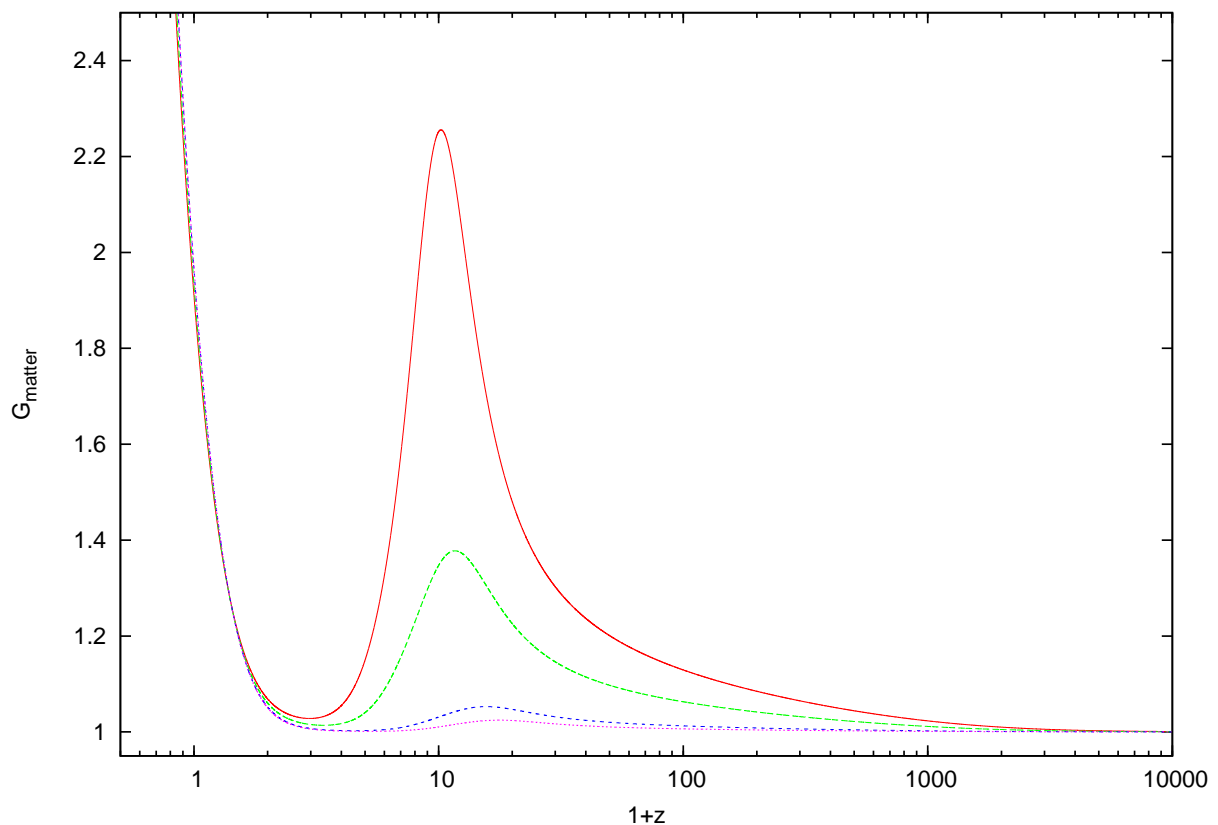


Figure 2. The gravitational strength evolution varies with the initial dark energy density. Here the curves have $\rho_{de}(z = 10^6) = 10^{-4}$, 5×10^{-5} , 10^{-5} , and 5×10^{-6} from top to bottom. While the bump in strength grows and shifts to later times for higher density, the early time and late time behaviors are on attractors. An interesting complementarity exists with the effective dark energy equation of state such that the less deviation from GR in G_{matter} , the stronger phantom deviation from $w = -1$.

runs the risk of substantially biasing both results.

Fortunately, simultaneous fitting is straightforward (with proper definition of parameters to separate the physics) and does not substantially degrade the constraints. Next generation observations will carry a wealth of information on the cosmic density and velocity fields, and enable model independent gravity constraints on 8 post-GR parameters at the 10% or better level. For the simplest extensions to general relativity this is within the precision necessary to distinguish the correct theory.

Galileon cosmology offers a more robust foundation with respect to physical naturalness. It also provides a rich phenomenology with attractor solutions in the early and late universe and multiple signatures in both the expansion and gravity behavior. The parameter space can be constrained through both theoretical considerations and comparison to observations. Although the exploration of the paradoxical topic of the direction of gravity is still in early days, the results so far are exciting – robust, testable alternatives to general relativity. The next generation of cosmology surveys and theoretical studies should finally test gravity diligently on cosmic scales and address the mystery of cosmic acceleration.

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